**RESERVE**

**Functionality of the Release of the Real-Time Solver**

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**Abstract**

This deliverable provides an overview of DPsim and its correspondent library. DPsim is a dynamic phasor solver for real-time simulation. The solver computes the solution of a network consisting of models defined in the dynamic phasor domain for every simulation step. This first version of the solver has been implemented in C++ and will be delivered as an open source solution. This deliverable describes the reference implementation of the solver along with the main algorithms adopted for the simulation and the models implemented in the framework. This deliverable also describes how the DPsim reference implementation is going to be shared among all the partners of the consortium.

**Keyword list**

Real-time simulation, dynamic phasors, modified nodal analysis, resistive companion, distributed co-simulation

**Disclaimer**

All information provided reflects the status of the RESERVE project at the time of writing and may be subject to change.
Executive Summary

The research of RESERVE touches fundamental concepts of today's energy systems. Decreasing bulk energy generation while increasing the share of distributed energy resources, has an impact on the entire system as a whole. The current practice of controlling the grid in a hierarchical way is being steadily replaced with a decentralized approach relying heavily on communication.

Testing new concepts in a large scale power system scenario is a very difficult and a challenging task for two reasons: cost and control of all system parameters. A test grid would have to be set up separately from the grid that provides energy to customers to ensure quality of service. Moreover, it is not possible to control all test parameters, for example, the weather which has a large impact considering the share of increase in renewable energy sources.

RESERVE proposes a distributed real-time simulation environment which relies on the internet and utilizes the simulation resources available from project partners to test the concepts developed in the project. Real-time simulation allows for the integration of hardware which enables tests that include real devices and software components. Given that commercial real-time simulators usually come at a high cost, RESERVE has progressed the development of a minimal real-time solver that can be used to connect distributed simulations. While this real-time solver does not work on the electromagnetic transient domain like commercial real-time solvers this means that the simulation is not as accurate. However, this does allow for more flexibility in the selection of the simulation time step, which is of advantage for distributed simulation. This report presents the basic functionalities of this solver called DPsim.

Before explaining the details of the solver, this report provides a technical analysis of the difficulties tied to internet distributed simulation. Then, we propose dynamic phasors as a solution that could be applied to improve the accuracy of the simulation results for large time steps. The time step is of particular interest since the communication delay in the range of tens of milliseconds between simulators interconnected through wide area networks is much larger than the simulation time step typically used in electromagnetic transient real-time simulations.

Results in the dynamic phasor and electromagnetic transient domain are compared to quantify the advantage of dynamic phasor simulations in practice. The test platform for this evaluation is the power system simulator, which is currently under development. It is shown that the dynamic phasor results are significantly better than the electromagnetic transient results for time steps of tens of millisecond which is in the range of the expected delays for distributed simulation in Europe.
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1. Introduction

Renewables in a Stable Electric Grid (RESERVE) is a three-year European Commission funded project within the Work Program H2020-LCE-2016-2017. The project officially started in October 2016.

1.1 Task 4.2

This deliverable is the first major output of Task 4.2 in WP4. This task is about the development of the open source real-time simulator DPsim, which is going to be developed in the frame of RESERVE. This deliverable describes the fundamental theory behind DPsim, whereas D4.3, the second version of this deliverable, will describe the implementation of more complex models in the solver and the connection of the VILLAS framework, which is described in D4.1.

1.2 Objectives of the Work Report in this Deliverable

While the primary objective of this report is to lay out the basics of DPsim, the laboratory interface design is touched on as well because of the strong interconnection between the two components.

1.3 Outline of the Deliverable

The first part of this report explains the challenges of distributed real-time simulation and presents previous work in this field. Then, the theory behind the developed real-time simulator is outlined. The main pillars of DPsim, modified nodal analysis, dynamic phasors and the resistive companion method are described. Finally, a small simulation example is presented which shows the effect of using dynamic phasors compared to the more traditional EMT approach.

1.4 How to Read this Document

This document can be read on its own, but should the reader want to learn about the components that interconnect real-time simulators such as the simulator described in this deliverable, we suggest reading deliverable D4.1. Overall, this deliverable (D4.2) is related to the following document from the RESERVE project:

- D4.1 – Demonstration of prototype of laboratory infrastructure
- D4.3 – Functionality of the releases of the real-time solver, V2
- D4.4 – First interconnection test of the nodes in pan-European simulation platform

D4.1 offers insight into the laboratory interconnection infrastructure, which is the framework that is connecting instances of DPsim and other real-time simulators. This deliverable (D4.2) is input to the second version D4.3 and explains the simulator that is used for first interconnection tests, which are described in deliverable D4.4. The chart below provides a graphical representation of the dependencies between the deliverables in WP4 of RESERVE.

![Chart of Deliverables](chart.png)

*Figure 1: Relations between deliverables in WP4 and other work*
2. Real-Time Simulation in RESERVE

The transition to energy systems, which will rely almost entirely on generation by renewable energy sources (RES), is inevitable and a global priority to tackle climate change. Traditional bulk generation must be decreased and energy systems need to be decentralized to support the integration of more RES thus requiring new concepts and techniques for ancillary services supporting voltage and frequency control. This transformation towards a decentralized system includes ICT to enable near real-time distributed decision-making, placing requirements on ICT which today’s systems do not meet. 5G ICT systems offer the prospect of seamless and secure connectivity, data processing with the reliability, availability and resilience required by energy providers and their customers in systems to be on the global market from 2020 onwards.

Real-time simulation is becoming increasingly popular to test and validate physical components and algorithms in a controlled and realistic environment. The synchronization of simulation time with wall clock time allows the exchange of physical inputs and outputs between externally connected devices and the real-time simulator since a second in real-time takes one second to be simulated. When a device is attached to a simulation, so-called Hardware-In-the-Loop (HIL), the behavior of components in the system can be safely tested also during emergency operations, and the integration of the device into the system can be easily validated. The applications of real-time simulation in power system analysis are, for example, protection and control system development and testing, distributed generation modeling, especially with renewable energy resource integration and microgrid control [1]. However, real-time simulators and the devices-under-test might be geographically distributed or the capabilities of locally available real-time simulators might not be sufficient for the given simulation scenario. Then, the model to be simulated can be partitioned for a distributed simulation [2].

One of the main objectives of RESERVE is the development of an innovative pan-European real-time, internet-distributed simulation infrastructure, which is able to support large scale experimental activity in the field of power dynamics and automation. This infrastructure will be a living roadmap supporting the decision-making process in the energy transition including new control algorithms and the increase of communication technology in power systems enabled by the new 5G standard.

The reasons for distributing the simulation are manifold. It allows for the sharing of available hardware and software in different real-time simulation laboratories among participants to enhance computational power and facilitate remote Software-In-the-Loop (SIL) and (Power-)Hardware-In-the-Loop (PHIL/HIL). Then, new devices that are going to be integrated in power systems can be tested even in remote locations where real-time simulation capabilities are not available. Large scale system simulations are facilitated because the real-time capabilities of several simulation facilities could be harnessed. Besides, confidential data does not need to be shared as each laboratory can be responsible for simulating its own part of the model locally, solely exchanging interface variables with other interconnected systems, imitating the real world where regional or national power grids are interconnected through tie-lines. The implementation of this distributed infrastructure can be broken down roughly into three subtasks: Interfacing the real-time simulation laboratories over a communications network, developing a simulation solver and integrating the infrastructure in a cloud-based interface supporting location agnostic access to the simulation.

The exchange of real-time power system simulation data poses a challenge in internet-distributed simulation since the communication delay between simulators may be orders of magnitude larger than the simulation step of commercial electromagnetic transient (EMT) real-time simulators. In this case, it is not possible to exchange data between the simulators in every time step and a compensation of the communication delay is required. Alternatively, the simulation could be based in the classic phasor domain, which allows larger simulation time steps. However, this would defy the objective of developing new frequency control algorithms since classic phasor domain simulations assume a fixed frequency. In RESERVE, the approach is to engage the problem in two ways. A novel co-simulation interface called VILLAS framework enables the real-time data exchange between laboratories. Furthermore, the real-time solver, which is subject to this report, allows an increase of the simulation step size while supporting variable frequency scenarios.
Figure 2 shows one of the possible scenarios that could be tested with the available components in WP 4. If the co-simulation model is a part of the Romanian transmission grid, UPB could provide measurement data from the Romanian grid. WIT uses the real-time solver developed in RESERVE to join the simulation while RWTH and POLITO employ a combination of their commercial EMT simulators and the new solver.

![Diagram of co-simulation scenario]

Figure 2: Example co-simulation scenario for WP 4

The first version of DPsim described in this report allows the simulation of networks that consist of ideal voltage and current sources, sources with internal impedance and basic passive components like resistors, capacitors and inductors. The code is maintained in a GitLab (https://about.gitlab.com/) instance hosted by RWTH. This platform will also be used to provide the solver to the project partners.
3. Challenges of Distributed Real-Time Simulation

Geographically distributed simulation borrows problems from parallel simulation. In both cases the model must be partitioned and it has to be assured that the parallel simulation results do not differ from the results of a sequential simulation. Furthermore, real-time requirements decrease the number of suitable simulation techniques [3]. Typically, power system real-time simulators follow the fixed time step approach since more sophisticated variable time step integration methods are not suitable for fast calculations as needed for real-time execution and small-time steps [5].

A common approach to partition power systems for parallel simulation is the use of travelling wave transmission line models [6]. However, it should be noted that electromagnetic waves travel about 15 km in 50 µs, which is a typical step time in real-time power system simulation. The expected delay in internet-distributed simulation is tens of milliseconds. In case of geographically large distances between the simulators, the time needed for information exchange can reach tens of milliseconds. Therefore, the insertion of a line with the required length into the model would have a severe impact on the behavior of the system.

Without compensation, the communication delay might cause large errors and even instability as shown in [1] for a delay of more than 10 ms. One cause for this is the sampling requirement imposed if an AC 50 Hz or 60 Hz system is simulated in EMT. According to the sampling theorem, the minimum sampling frequency is twice the maximum frequency expected in the system. This combined with the large RTT expected in geographically distributed simulations, complicates the synchronization among simulators.

Simulations using traditional static phasors do not impose the strong sampling requirement since the system frequency is implicitly included but this frequency is fixed. Therefore, this approach does not support frequency control or stability studies, for example, on transmission level. A more detailed explanation of the difference between static and dynamic phasors can be found in the Appendix Section 0 while the general theory of dynamic phasors is covered in Section 4.3.

The authors of [7] realized an integrated real-time co-simulation laboratory by applying a communication platform as a simulator-to-simulator interface proposed in [1] in order to enable remote and online monitoring of an interconnected transmission-distribution system. Based on that novel approach, each simulator carried out simulations in time domain, while the time-varying Fourier coefficients of the quantities in the interconnection node, i.e. decoupling point, are exchanged. In the model, the interface was represented as ideal transformer model (ITM) [8].

![ITM represented by current and voltage source](image)

Figure 3: ITM represented by current and voltage source

EMT values could not be exchanged for every simulation step due to the communication RRT. Apart from that, the local EMT simulation is computational less efficient compared to phasor simulations. Instead, following transformed quantities could be exchanged: static phasors or dynamic phasors of the fundamental and harmonic components. The delay is compensated by shifting the phasors in time at the receiving end.

The problem with the former solution is the following. For transient analysis, frequency deviation in one side cannot be captured on the other side to perform a distributed real-time simulation with the same results as a local real-time simulation. With the latter solution, dynamic phasor exchange, simulations do not imply fixed system frequency.
The approach explained in [7] requires the extraction of phasor information from the EMT signals. Therefore, transparency of the interface is not given since the interface algorithm may alter the exchanged signals. So far, the interface algorithm can extract magnitude and phase for several harmonic components. The frequency is assumed to be the nominal system frequency with a DC link connecting the two systems.

3.1 Summary

The following list summarizes the problems explained above:

- In internet distributed simulation, RRT between simulators of several tens of ms
- If EMT values are exchanged among simulators, a time step of 10 ms (50 Hz AC) or smaller is required according to the sampling theorem
- Variable system frequency is not support by classic static phasors
- EMT-Phasor-Interface requires extraction of the phasor information and may alter simulation data

Section 4 describes the real-time solver under development which is supposed to avoid / cope with these limitations.
4. Basic Structure of the Real-Time Solver

Typically, geographically distributed simulators are connected via Wide Area Networks (WAN) based on VPNs. In addition to the challenge of partitioning the model and creating interfaces for parallel execution, the communication delay can be even larger than the simulation time step which is typically used in power system EMT simulations. The real-time solver described in this section aims at increasing the simulation time step to close or at least minimize this large between step size and communication delay and avoid the need for an extraction of the phasor information before sending them to other simulators.

The base of the solver is the nodal analysis as it is the case for many commercial EMT solvers. In addition to this, resistive companion models are introduced to increase the number of component types that can be directly solved with this method. The novel part about the solver is the use of dynamic phasors for the representation of state variables instead of classic phasors or electromagnetic transient variables.

The following three subsections provide an overview of these three concepts and how they are employed in the solver.

4.1 Nodal Analysis

The nodal analysis method provides a schematic way to find a set of equations that fully represent an electric circuit consisting of current sources and resistances. Therefore, it is a common algorithm used in circuit simulation software. This section describes the general procedure.

A circuit with \( b \) branches has \( 2b \) unknowns since there are \( b \) voltages and \( b \) currents. Hence, \( 2b \) linear independent equations are required to solve the circuit. If the circuit has \( n \) nodes and \( b \) branches, it has

- Kirchoff’s current law (KCL) equations
- Kirchoff’s voltage law (KVL) equations
- Characteristic equations (Ohm’s law)

The nodal analysis method reduces the number of equations that need to be solved simultaneously. \( n - 1 \) voltage variables are defined and solved, writing \( n - 1 \) KCL based equations. A circuit can be solved using nodal analysis, by performing the following steps:

- Step 1: Select a reference node (mathematical ground) and number the remaining \( n - 1 \) nodes, that are the independent voltage variables.
- Step 2: Represent every branch current \( i \) as a function of node voltage variables \( v \) with the general expression \( i_k = g(v) \).
- Step 3: Write \( n - 1 \) KCL based equations in terms of node voltage variables. The resulting equations can be written in matrix form and must be solved for \( v \).

In matrix form, the equation to be solved can be written

\[
G \cdot x = A
\]  

(1)

where \( x \) is the vector of unknown variables, \( G \) is the system matrix and \( A \) is the vector of known sources. The advantage of writing the equations in matrix form is that the matrix can be populated in a very systematic way. For example, a resistance that is connected to nodes \( j \) and \( k \) can be considered by stamping the matrix of the complete system in rows and columns \( j \) and \( k \) as follows:

\[
G = \begin{bmatrix}
\vdots & \vdots & \vdots \\
1 & \frac{1}{R} & \frac{1}{R} \\
\frac{1}{R} & \frac{1}{R} & \frac{1}{R} \\
\vdots & \vdots & \vdots
\end{bmatrix}
\]  

(2)

An ideal current source on the other hand does not alter the system matrix but the right-hand side vector of the equation:
\[ A = \begin{bmatrix}
    \vdots & \vdots & \vdots \\
    j & 1 & -1 \\
    k & -1 & \vdots \\
    \vdots & \vdots & \vdots 
\end{bmatrix} \]  

This method is limited to resistances and cannot represent other passive components like capacitances and inductances.

### 4.2 Resistive Companion Method

The nodal analysis method is further extended by resistive companion models. Components that store energy, such as inductors and capacitors require some kind of “memory” of simulation steps in the past. Applying the trapezoidal integration rule to the following equation which describes an inductor

\[ \frac{d}{dt} i(t) = \frac{1}{L} v(t) \]  

results in

\[ i(k+1) = i(k) + \frac{\Delta t}{2L} (v(k) + v(k+1)) \]  

where \( \Delta t \) is the simulation time step. Equation (5) can be visualized as in Figure 4.

![Figure 4: Resistive companion transformation of inductor](image)

At this point, the matrix stamp presented in Section 4.1 can be utilized again since the resulting model consists only of a current source and a resistance. The same approach can be applied to capacitances as shown in the appendix.

### 4.3 Dynamic Phasors

In commercial real-time solvers, the nodal analysis method is employed with time domain variables or classic phasors as shown in Section 4.1 and 4.2. As mentioned before, the aim of the dynamic phasor solver is to take advantage of the phasor representation while maintaining the variable frequency feature of EMT simulations. Furthermore, an increased step size is not only desirable for distributed simulation; it also facilitates the calculation of larger grids on one computation node.

Dynamic phasors were initially developed for power electronics analysis [9]. Later, the concept was extended to power systems analysis [10]. Subsequently, the authors in [11] describe the use of dynamic phasors for power system simulation. Besides, they are still used to construct efficient models for the dynamics of switching gate phenomena with a high level of detail to simulate the integration of new DERs and HVDC converter technologies [12] [13]. Fault analysis and unbalanced conditions are other important research topics in which dynamic phasors allow larger models and simulations that are more efficient. Asymmetrical faults are studied in [14] [15] [16] [17]. In the last two articles, the behavior of AC machines like Doubly-Fed Induction Machine (DFIG) wind turbines or synchronous generators are evaluated using dynamic phasors. In [18] an effort to generalize the dynamic study with dynamic phasors is made by modeling and validating a multiple synchronous generator test grid. The goal was twofold: Application of dynamic phasors for multi-source, multi-frequency systems and modeling of systems with time-varying frequencies.

Although dynamic phasors allow a significant saving in terms of computational cost, the idea to apply the concept to real-time simulation is still fairly novel. Commercial simulators like RTDS and OPAL-RT offer analytic modeling tools being able to perform EMT simulations e.g. eMEGAsim
developed by OPAL-RT. OPAL-RT introduced also simulation tools in the traditional complex phasor domain called ePHASORsim, which is limited to system fundamental frequency. Dynamic phasors might allow larger systems to be simulated in real-time with larger time-steps (i.e. milliseconds instead of microseconds) while catching the dynamic behavior of a system with frequency deviation.

As mentioned in Section 3, the exchange of time-domain values among simulators imposes strong requirements on the sampling rate. Therefore, previous work already introduced dynamic phasors as a means of exchanging data in the frequency domain rather than the time domain [19]. However, this requires the extraction of the dynamic phasors from the time domain signal or every simulation step. Instead, we propose to simulate the entire system in dynamic phasors to be able to increase the simulation time step and to avoid the conversion from the time domain to the frequency domain.

In the following, the general approach of dynamic phasors for power system simulation is explained while pointing out the main features that are interesting for the real-time solver and distributed simulation. Using dynamic phasors, it is possible to treat an AC signal as a DC signal without losing its dynamic properties as it is the case when using static phasors in power system analysis. Instead of fixing the frequency, the signal is shifted by the system frequency, e.g. 50 Hz. Besides, one time domain variable can be approximated by several dynamic phasors of different harmonics, each of these shifted by their center frequency. However, this shift only decreases the maximum frequency of the simulated signals if all frequencies of interest lie in a small band around these center frequencies. The fundamental frequency of power systems is normally varying in a region close to the nominal system frequency. Hence, the bandpass limitation is fulfilled. The shift in the frequency domain for a bandpass signal represented in grey is visualized in Figure 5. The signal is shifted from the center frequency \( \omega_c \) to \( \omega = 0 \).

The bandpass signal \( X_{bp} \), centered around \( \omega_c \), is real valued and can be represented in the right half plane of the frequency spectrum. The shifted signal, which is also denoted baseband signal, \( X_{bb} \), features a smaller maximum frequency. According to the sampling theorem, the baseband signal requires a smaller sampling rate to be represented correctly. This property is very important in the application of real-time simulation since the RTT between two simulators in different locations can be very significant. In case of pan-European simulations, the RTT has been found to be several tens of ms [20], whereas links between Europe and the US can exhibit a RTT of well over 100 ms [1]. Therefore, the default time step of 50 \( \mu \)s, used by many commercial real-time simulators, does not allow a data exchange between the simulators for every simulation step without compensation for the communication delay.

In the following, the general dynamic phasor approach is explained which is the basis of the simulation example in the next section. First, the time domain signal \( x \) is approximated with a Fourier series representation:

\[
x(\tau) = \sum_{k} X_{k}(t)e^{j\omega_{k}\tau}
\]

where \( \tau \in (t - T, t] \). The \( k^{th} \) coefficient is determined by

![](image.png)

**Figure 5: Frequency shift of dynamic phasors**
\[ X_k(t) = \langle x \rangle_k(t) = \frac{1}{T} \int_0^T x(t) e^{-j k \omega_s(t)} dt \]  

(7)

where \( \omega_s \) is the fundamental system frequency and \( k \omega_s \) are its harmonics. Deriving equation (7) leads to

\[ \frac{d}{dt} \langle x \rangle_k(t) = \left\{ \frac{d}{dt} \langle x \rangle_k(t) \right\} - j k \omega_s \langle x \rangle_k(t) \]  

(8)

Accordingly, a state space model of the general form

\[ \frac{d}{dt} x(t) = f(x(t), u(t)) \]  

(9)

would be transformed to the equation given in (10).

\[ \frac{d}{dt} \langle x \rangle_k(t) = \left\{ f(x(t), u(t)) \right\} - j k \omega_s \langle x \rangle_k(t) \]  

(10)

Applying (10) to the equation of an inductance

\[ \frac{d}{dt} i(t) = \frac{1}{L} \cdot v(t) \]  

(11)

results in the following equation for the fundamental dynamic phasor:

\[ \frac{d}{dt} \langle i \rangle_1(t) = \frac{1}{L} \cdot \langle v \rangle_1(t) - j \omega_s \langle i \rangle_1(t) \]  

(12)

4.4 Summary

This section describes the concepts which are combined in the development of the real-time solver, DPsim: Nodal analysis, resistive companion method and dynamic phasors. The base of the solver is the representation of the circuit according to the nodal analysis which is extended by the resistive companion method. Instead of using the real voltage and current variables, DPsim calculates in the dynamic phasor domain.

In Section 5, a simple circuit, that includes an inductance modeled according to the approach described previously, is simulated using the EMT and dynamic phasor approach for different time steps.
5. Simulation Results Using the Real-Time Solver

To support the theoretical advantage of dynamic phasor over EMT simulations, we present the simulation results for a simple circuit as depicted in Figure 6. The circuit consists of an AC voltage source of $V_{Source} = 1kV$ peak voltage with a resistance of $R_{Source} = 1\Omega$, a RX-series element of $R_{Line} = 1\Omega$ and $L_{Line} = 100mH$ and a load resistance of $R_{Load} = 100\Omega$. Internally, the voltage source is transformed to its Norton equivalent.

The simulation scenario is as follows. At 0.2s, the load resistance is decreased to 50 $\Omega$ and at 0.4s the frequency of the AC voltage source is decreased from 50 Hz to 45 Hz. This scenario is simulated for different time steps between 50 $\mu$s and 40 ms using the real-time solver described before. The EMT simulations are based on the traditional approach using real valued variables instead of dynamic phasors. In the following, we compare the voltage $V_{Load}$ across the load resistance.

![Figure 6: Example circuit for the comparison of dynamic phasor and EMT simulations](image)

As can be seen in Figure 7 and Table 1, the results are almost identical for time steps of 50 $\mu$s. Figure 7 shows the EMT results, the absolute value of the fundamental dynamic phasor and the time domain signal of the fundamental dynamic phasors after it is shifted back by 50 Hz in the frequency domain.

![Figure 7: Comparison of dynamic phasors and EMT simulation for time steps of 50 $\mu$s](image)

<table>
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<th>DP</th>
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<td>0.05</td>
<td>0</td>
<td>-</td>
<td>3.97E-05</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>105.61</td>
<td>138.92</td>
<td>97.004</td>
<td>110.75</td>
</tr>
<tr>
<td>5</td>
<td>7912.6</td>
<td>21133</td>
<td>2086.7</td>
<td>588.39</td>
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<tr>
<td>10</td>
<td>2.53E+05</td>
<td>2.24E+05</td>
<td>5764.8</td>
<td>914.98</td>
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<tr>
<td>15</td>
<td>3.84E+05</td>
<td>4.58E+05</td>
<td>12080</td>
<td>2050.3</td>
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</table>
The shift and transformation into the time domain is accomplished by taking the real part of the signal after applying equation (6).

\[ x_{re}(\tau) = Re \left\{ \sum_{k} X_k(t) e^{jk\omega_s(\tau)} \right\} \]  

(13)

Furthermore, Table 1 depicts the mean squared error for the signals after linear interpolation. It is important to point out that the interpolation of the dynamic phasors is applied for real and imaginary part separately and before shifting the signal back to 50 Hz. Comparing the 20 ms time step results presented Figure 8 and Figure 9, it can be seen that the dynamic phasor simulation is very accurate even for large time steps. Without interpolation, the fundamental sinusoidal is not represented correctly by the dynamic phasor values since the number of data points is too small.

![Figure 8: Comparison of dynamic phasors and EMT simulation for time steps of 20 ms](image)

![Figure 9: Comparison of dynamic phasors and EMT simulation for time steps of 20 ms with interpolation](image)

From Table 1, it can be concluded that the error is growing much slower for dynamic phasor simulations. Tens of ms seem to be feasible time steps for dynamic phasor simulations if the system transients are not too fast. Therefore, the dynamic phasor approach could enable
distributed simulations without having to compensate for the communication delay in some cases, for example, distributed simulation among participants in Europe.
6. Conclusion

The first part of task 4.2 is the identification of challenges posed by our use case, the pan-European real-time simulation, and research on how these challenges were tackled in previous work. After presenting the results of this investigation, dynamic phasors are proposed as solution to solve one of the main problems of distributed real-time simulation, the large round trip time between the simulators which determines the minimum time step if the simulators are to exchange data for every step. Furthermore, a larger time step is beneficial for large system simulations.

The three main methods used to build the real-time solver are explained and we present a study that shows the advantage of using dynamic phasors for distributed real-time simulation. The focus of this study is on the simulation time step. The larger the time step, the lesser the impact of the communication delay between two geographically distributed simulators on the real-time data exchange.

Currently, the connection to the co-simulation interface which is being developed in task 4.1 is added to the dynamic phasor solver presented here. As next step, we plan to investigate its capabilities regarding parallel power system simulation.
7. References


8. List of Abbreviations

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<td>Electromagnetic Transient</td>
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<tr>
<td>DFIG</td>
<td>Doubly-Fed Induction Machine</td>
</tr>
<tr>
<td>ITM</td>
<td>Ideal Transformer Model</td>
</tr>
<tr>
<td>HIL</td>
<td>Hardware-In-the-Loop</td>
</tr>
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<td>RES</td>
<td>Renewable Energy Sources</td>
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<tr>
<td>RTT</td>
<td>round-trip time</td>
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<td>Software-In-the-Loop</td>
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Annex

A.1 Dynamic Phasor Equations for Resistive Companion Capacitor

The equation of a capacitor is given in (14).

\[
\frac{d}{dt} v(t) = \frac{1}{C} \cdot i(t)
\]  

(14)

Hence, the equation for the fundamental dynamic phasor can be derived:

\[
\frac{d}{dt} (v)_1(t) = \frac{1}{C} \cdot (i)_1(t) - j \omega_s (v)_1(t)
\]  

(15)

Figure 10: Resistive companion model of a capacitor

The resistive companion model is calculated from (15) using the trapezoidal rule which results in (16).

\[
\begin{align*}
    i_1(k + 1) &= \frac{1 + jb}{a} \cdot v_1(k + 1) - \frac{1 - jb}{a} \cdot v_1(k) - i_1(k)
\end{align*}
\]  

(16)

A.2 Dynamic Phasors and Static Phasors

Dynamic phasors may be time variant whereas traditional static phasors are time invariant as depicted in Equations (17) and (18).

\[
\begin{align*}
    x_{dph}(\tau) &= \Re \left\{ \sum_k X_{dph,k}(t) \cdot e^{jk\omega_s(\tau)} \right\} \\
    x_{cph}(\tau) &= \Re \left\{ \sum_k X_{cph,k} \cdot e^{jk\omega_s(\tau)} \right\}
\end{align*}
\]  

(17)

(18)

This means that the solution of the grid is independent from any previous solution in time in the case of static phasors. Therefore, dynamic phasors may represent a signal whose frequency spectrum is dispersed around harmonics of the fundamental frequency \(\omega_s\), while the static phasor only describes the behavior of the system at the fundamental frequency. This property is visualized in Figure 11 and Figure 12. Figure 11 shows three different signals, a 50 Hz sinusoidal and two modulated versions, amplitude and frequency modulation, where the 50 Hz sinusoidal is used as carrier signal. Here, the static phasor approach would only capture the amplitude of the
50 Hz carrier signal while the dynamic phasor can also describe the amplitudes of the other frequencies present in the modulated signals.

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**Figure 11: Carrier signal and modulated signals**

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**Figure 12: Frequency spectrum of carrier signal (a), amplitude modulated signal (b) and frequency modulated signal (c)**